Series Solution

in

Homogeneous second order linear Differential Equation with variable coefficients.

$$\boxed{P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)\,y = 0} \quad \text{---}(1)$$

Here:-

$P_0(x)$, $P_1(x)$ & $P_2(x)$ $\rightarrow$ Polynomials in Powers of x.

Homogeneous $\rightarrow$ R.H.S zero $\rightarrow$ No terms of x only.

Second order $\rightarrow$ Second Derivative

Linear DE $\rightarrow$ Max Power of y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ $\rightarrow$ one.

Variable coefficients $\rightarrow$ $\because$ P_0, P_1 & P_2 are Polynomials in x

From Eq. (1):-

$$\frac{d^2y}{dx^2} + \frac{P_1(x)}{P_0(x)}\frac{dy}{dx} + \frac{P_2(x)}{P_0(x)}y = 0$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0} \quad \text{— (2)}$$

Eq. (2) is called Normal form/Canonical form/Standard form of Homogeneous Second order Linear DE with Constant Coefficients.

Here:-

$$P(x) = \frac{P_1(x)}{P_0(x)}$$

$$Q(x) = \frac{P_2(x)}{P_0(x)}$$

lassification of Point $x = a$:-

Point : $x = a$

- **Ordinary Point (OP)**
  If :- at $x = a$
  $P_0(a) \neq 0$

- **Singular Point (SP)**
  If :- at $x = a$
  $P_0(a) = 0$

  - **Regular Singular Point (RSP)**
    If :- both
    $$\lim_{x \to a} (x-a)\frac{P_1(x)}{P_0(x)} = \text{Finite}$$
    $$\lim_{x \to a} (x-a)^2\frac{P_2(x)}{P_0(x)} = \text{Finite}$$

  - **Irregular Singular Point (ISP)**
    Otherwise

Note:-

1) Finite or Analytic

2) $\frac{P_1(x)}{P_0(x)} = P(x)$

3) $\frac{P_2(x)}{P_0(x)} = Q(x)$